Quantum Mechanics as a Classical Theory IV: The Negative Mass Conjecture

L. S. F. Olavo
Departamento de Fisica - Universidade de Brasilia - UnB
70910-900 - Brasilia - D.F.- Brazil

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Abstract

The following two papers form a natural development of a previous series of three articles on the foundations of quantum mechanics; they are intended to take the theory there developed to its utmost logical and epistemological consequences. We show in the first paper that relativistic quantum mechanics might accommodate without ambiguities the notion of negative masses. To achieve this, we rewrite all of its formalism for integer and half integer spin particles and present the world revealed by this conjecture. We also base the theory on the second order Klein-Gordon's and Dirac's equations and show that they can be stated with only positive definite energies. In the second paper we show that the general relativistic quantum mechanics derived in paper II of this series supports this conjecture.

1 General Introduction

What is the job of a theoretical physicist? The first answer that comes to us in a somewhat precipitate manner is: The theoretical physicist's job is to say how the world is. Despite the obvious philosophical fragility of such an assertion, hardly adjustable with the method of systematic doubt of science, this answer gives us a key for a more adequate approach. The theoretical physicist has not the mission of saying how the world is but, rather, the job to explain how the world might be. Only the experiments have the final word about, among all the numerous possible worlds furnished by a theory, which one is more adequate. The history of science of the last four centuries has shown that we shall not underestimate any of the models we uncover with our interpretations of the underlying formal apparatus.

This is the spirit underlying the first paper of this series. In this paper, we will show that relativistic quantum mechanics admits an interpretation very

different from the one usually accepted. Since we are not interested in obtaining a new relativistic quantum mechanical formalism, its formal apparatus will be kept intact. Our interest is to uncover another world, equally allowed by this apparatus, and show that an arbitrary choice has hidden this world.

The methodological criterion that one should apply in judging the merits of this work cannot be related to its applicability, since we keep the pure formal apparatus intact and expect the same formal outcomes. This criterion has to do with the world picture that emerges from one and the other theoretical interpretations. It is from this point that they become different theories and expect answers from Nature so distinct as excluding. The corroboration of one choice or the other is left, however, for the experiments...

In the second section of this paper, we will introduce, in a rather intuitive way, the main idea of this first paper. We will claim that the relativistic quantum mechanical formalism can accommodate a world with negative masses. We will use the Klein-Gordon theory in elaborating such an argument.

In the third section, we will develop the considerations made in the previous one into a more mathematical format.

We will make, in the fourth section, an extension of the previous formalism to apply it to particles with half integral spin. We will then use the second order Dirac's equation.

After the fourth section, the first part of this series of two papers will be complete. We will have demonstrated that all relativistic quantum mechanics can be rewritten to accommodate negative masses. We then make our conclusions.

We devote the appendix to show that relativistic quantum mechanics based on second order equations can be rewritten to admit only positive probability densities. We then show that the solution of this problem bears some resemblance with the formalism developed in the main text.

The next paper is a continuation of the first. We make an application of the general relativistic quantum mechanical formalism already derived[1, 2, 3] to a simple, but highly instructive, example.

In the second section of that paper, we apply the formalism to the simple problem of a test mass gravitating around a heavy body (we call this problem the quantum Schwartzchild problem). From the results so obtained, we show that this general relativistic quantum theory supports the negative mass conjecture of the first paper.

In the third section we make our final conclusions.

2 Introduction

When the Klein-Gordon theory (hereafter KG) was proposed, the possibility of negative probability densities was one of its main deficiencies. The solution met was to multiply this density by the modulus of the electric charge and to consider

it as a charge, rather than a probability, density. This attitude, however, seems to be based on an arbitrary choice that has hidden other possibilities.

The usual interpretation of the relativistic quantum mechanical formalism assumes, by principle, that there can be no negative masses in Nature[4, 5]. We now turn to show that we can eliminate this constraint from the interpretation without incurring into inconsistencies.

In a previous series of papers, we have shown that the KG probability density, defined as

$$j_0(x) = \frac{i\hbar}{2mc}\left(\phi^*(x)\,\partial_0\phi(x) - \phi(x)\,\partial_0\phi^*(x)\right), \tag{1}$$

where m is the particle's mass and ϕ is the associated probability amplitude, does not require to be multiplied by any charge to represent a true probability density if we accept that we should have negative masses for antiparticles. We then could write the probability density as

$$\rho_\lambda(x) = \frac{i\hbar}{2\lambda mc}\left[\phi^*(x)\,\partial_0\phi(x) - \phi(x)\,\partial_0\phi^*(x)\right]_+ \ , \tag{2}$$

where $[\ ]_+$ implies that we take only the positive signal of the quantity inside brackets, and the parameter λ defines if the density refers to particles or antiparticles:

$$\lambda = sign\left(\phi^*(x)\,\partial_0\phi(x) - \phi(x)\,\partial_0\phi^*(x)\right) = \begin{cases} +1 & \text{for particles} \\ -1 & \text{for antiparticles} \end{cases} . \tag{3}$$

We might thus interpret a negative probability density as a positive one describing negative mass particles (antiparticles). In such a case, the mass distribution can be written as:

$$\rho_\lambda^{mass}(x) = m\rho_\lambda(x) = \frac{i\hbar}{2\lambda c}\left[\phi^*(x)\,\partial_0\phi(x) - \phi(x)\,\partial_0\phi^*(x)\right]_+ \tag{4}$$

and will be positive for particles and negative for antiparticles. From the very definition of the parameter λ it is easy to see that the complex conjugation, defined by $\phi \to \phi^*$, implies $\lambda \to -\lambda$ and thus, in the mapping of particles into antiparticles.

We can handle with electromagnetic fields in a way similar to the usually done in the literature. We then have

$$\rho_\lambda(x) = \frac{i\hbar}{2\lambda m_0 c}\left[\phi^*(x)\,\partial_0\phi(x) - \phi(x)\,\partial_0\phi^*(x)\right]_+ - \frac{2e}{m_0c^2}\Phi(x)\,\phi^*(x)\,\phi(x) =$$

$$= \frac{i\hbar}{2\lambda mc}\left[\phi^*(x)\,\partial_0\phi(x) - \phi(x)\,\partial_0\phi^*(x)\right]_+ - \frac{2\lambda e}{\lambda mc^2}\Phi(x)\,\phi^*(x)\,\phi(x)\,, \tag{5}$$

where Φ is the scalar electromagnetic potential. Collecting terms we get

$$\rho_\lambda(x) =$$

$$= \frac{1}{2\lambda mc^2}\left\{\left[\phi^*(x)\, i\hbar\frac{\partial\phi(x)}{\partial t} - \phi(x)\, i\hbar\frac{\partial\phi^*(x)}{\partial t}\right]_+ - 2\lambda e\Phi(x)\,\phi^*(x)\,\phi(x)\right\}, \tag{6}$$

which shows that the complex conjugation of the amplitudes also implies in the change of the electric charge (we can also see this looking directly at the KG equation). We conclude that, in the present theory, the complex conjugation operation has the effect of changing the mass and charge signs. This implies that particles shall have these properties with the opposite sign of the associated antiparticles.

We know from the experiments that particle-antiparticle pairs, when subjected to homogeneous magnetic fields, move along opposite circular trajectories; this is the reason for the usual interpretation considering the charge of particles and antiparticles to have opposite sign. In the present interpretation, both mass and charge change sign; thus, the ratio e/m does not change its sign.

It is important to stress, however, that the charge and the mass appear in the expression for the trajectory of the pair together with the velocity of its components. We now turn to see what happens with these velocities in our formalism.

Let us consider then the free particle-antiparticle solutions:

$$\phi_\lambda(x) = \exp\left[-\lambda i\left(E_p t - \mathbf{p}\cdot\mathbf{r}\right)/\hbar\right], \tag{7}$$

where the evolution parameter E_p and the momentum $\mathbf{p}$ are given by

$$E_p = mc^2/\sqrt{1-v^2/c^2} \quad ; \quad \mathbf{p} = m\mathbf{v}/\sqrt{1-v^2/c^2}. \tag{8}$$

The probability density and flux, in the absence of electromagnetic fields, are given by

$$\rho_\lambda(x) = \frac{E_p}{\lambda mc^2} \quad ; \quad \mathbf{j}_\lambda(x) = \frac{\mathbf{p}}{\lambda mc}. \tag{9}$$

If we put

$$\frac{\mathbf{p}}{m} = \mathbf{v} \quad \Rightarrow \begin{cases} \mathbf{p}_a = (-m)\,\mathbf{v}_a \\ \mathbf{p}_p = (+m)\,\mathbf{v}_p \end{cases}, \tag{10}$$

where $\mathbf{v}_p$ and $\mathbf{v}_a$ are the velocities of the particle and antiparticle, respectively, and $\mathbf{p}_p$, $\mathbf{p}_a$ their momenta, we then get

$$\mathbf{j}_\lambda(x) = \lambda\frac{\mathbf{v}}{c}, \tag{11}$$

which can be interpreted as meaning that the flux of particles in one direction is equivalent to the flux of antiparticles in the opposite direction. In this manner, we expect that, when gravitational forces are present, particles and antiparticles behave in the way shown in figure 1. These forces obviously do not pertain to the framework of the present theory; a theory that takes gravitation into account will be dealt with in the next paper. However it is noteworthy that particles and

antiparticles will not respond perversely to homogeneous magnetic fields as one could in principle think[6]. Indeed, when electromagnetic fields are present and taking $+e$ and $-e$ as the particle's and the antiparticle's charges, respectively, we have:

$$\rho_\lambda (x) = \frac{1}{\lambda mc^2} (E_p - \lambda e\Phi) \quad ; \quad \mathbf{j}_\lambda (x) = \frac{1}{\lambda mc} \left(\mathbf{p} - \lambda \frac{e}{c} \mathbf{A}\right), \tag{12}$$

which gives, for the flux

$$\mathbf{j}_\lambda (x) = \frac{1}{c} \left(\lambda \mathbf{v} - \frac{e}{mc} \mathbf{A}\right). \tag{13}$$

This shows that particles and antiparticles have velocity vectors with opposite signs compared to the potential vector. This property is sufficient to explain their behavior under the influence of a homogeneous magnetic field (figure 2).

We now proceed, in the next two sections, to rewrite the mathematical apparatus to state formally our conjecture.

3 Klein-Gordon's Theory with Negative Mass

If we depart from the hypothesis that Nature can reveal entities with masses of both signs, we then expect to find in It all the combinations shown in table I. We might use the Feshbach-Villars decomposition to relate all the possibilities furnished by nature with the KG equation. By means of this decomposition, the KG equation

$$\frac{1}{c^2} \left(i\hbar \frac{\partial}{\partial t} - e\Phi\right)^2 \varphi = \frac{1}{2m} \left(\frac{\hbar}{i} \nabla - \frac{e}{c} \mathbf{A}\right)^2 \varphi + m^2 c^2 \varphi, \tag{14}$$

when we use

$$\varphi_0 (\mathbf{r}, t) = \left[\frac{\partial}{\partial t} + \frac{ie}{\hbar} \Phi (\mathbf{r}, t)\right] \varphi (\mathbf{r}, t), \tag{15}$$

and

$$\varphi_1 = \frac{1}{2} \left[\varphi_0 + \frac{i\hbar}{m_0 c^2} \varphi\right] \quad ; \quad \varphi_2 = \frac{1}{2} \left[\varphi_0 - \frac{i\hbar}{m_0 c^2} \varphi\right], \tag{16}$$

becomes the following system of equations

$$\left[i\hbar \frac{\partial}{\partial t} - e\Phi\right] \varphi_1 = \frac{1}{2m} \left[\frac{\hbar}{i} \nabla - \frac{e}{c} \mathbf{A}\right]^2 (\varphi_1 + \varphi_2) + mc^2 \varphi_1; \tag{17}$$

$$\left[i\hbar \frac{\partial}{\partial t} - e\Phi\right] \varphi_2 = \frac{-1}{2m} \left[\frac{\hbar}{i} \nabla - \frac{e}{c} \mathbf{A}\right]^2 (\varphi_1 + \varphi_2) - mc^2 \varphi_2; \tag{18}$$

together with their complex conjugate

$$\left[i\hbar \frac{\partial}{\partial t} + e\Phi\right] \varphi_1^* = \frac{-1}{2m} \left[\frac{\hbar}{i} \nabla + \frac{e}{c} \mathbf{A}\right]^2 (\varphi_1^* + \varphi_2^*) - mc^2 \varphi_1^*; \tag{19}$$

$$\left[i\hbar\frac{\partial}{\partial t}+e\Phi\right]\varphi_2^*=\frac{1}{2m}\left[\frac{\hbar}{i}\nabla+\frac{e}{c}\mathbf{A}\right]^2(\varphi_1^*+\varphi_2^*)+mc^2\varphi_2^*. \tag{20}$$

With the notation above we note that it is possible to make a connection between the amplitudes and the particles signs of mass and charge they represent

$$\varphi_1 \Longleftrightarrow (+,+)\ ;\ \varphi_2 \Longleftrightarrow (-,+)\,, \tag{21}$$

$$\varphi_1^* \Longleftrightarrow (-,-)\ ;\ \varphi_2^* \Longleftrightarrow (+,-)\,, \tag{22}$$

where (A,B) represents an entity with mass and charge signs A and B, respectively.

We made the *choice*, in the last section, to represent antiparticles with the signs of the mass and charge reverted as related to the particle. In agreement with this choice, we shall attribute for pairs of such entities an amplitude and its complex conjugate, as become clear from equations (17-20).

We can now define the two-component spinors

$$\Psi=\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\quad;\quad\Psi^*=\begin{pmatrix}\varphi_1^*\\ \varphi_2^*\end{pmatrix}, \tag{23}$$

together with the Pauli matrices

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}\ ;\ \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\ ;\ \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix} \tag{24}$$

and rewrite the system (17-20) as

$$\left(i\hbar\frac{\partial}{\partial t}-e\Phi\right)\Psi=\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}\right)^2(\sigma_3+i\sigma_2)+mc^2\sigma_3\right]\Psi, \tag{25}$$

or else

$$\Psi_{c1}\left(i\hbar\frac{\partial}{\partial t}+e\Phi\right)=\Psi_{c1}\left[\frac{-1}{2m}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}\mathbf{A}\right)^2(\sigma_3+i\sigma_2)-mc^2\sigma_3\right], \tag{26}$$

where

$$\Psi_{c1}=\Psi^\dagger\sigma_3. \tag{27}$$

Using the basis

$$\mathbf{e}_1=\begin{pmatrix}1\\0\end{pmatrix}\ ;\ \mathbf{e}_2=\begin{pmatrix}0\\1\end{pmatrix}, \tag{28}$$

we can adopt the convention

$$u_0^{(P,+)}, \tag{29}$$

where the index zero indicates that we are in the inertial frame of reference and the dyad $(P,+)$ implies that the related spinor describe a particle with positive charge. It is then possible to write the four possible functions (Table 1) as

$$u_0^{(P,+)} = \mathbf{e}_1 e^{-iE_p\tau/\hbar}\ ;\ u_0^{(A,-)} = \mathbf{e}_1 e^{+iE_p\tau/\hbar}; \tag{30}$$

$$u_0^{(A,+)} = \mathbf{e}_2 e^{+iE_p\tau/\hbar}\ ;\ u_0^{(P,-)} = \mathbf{e}_2 e^{-iE_p\tau/\hbar}; \tag{31}$$

where

$$E_p = mc^2. \tag{32}$$

We might still define a charge conjugation by the operation

$$\Psi_c = \sigma_1 \Psi^*, \tag{33}$$

that satisfies a KG equation with the same mass sign but with the charge sign reverted. This spinor, however, cannot be now a candidate to represent antiparticles related to Ψ since only the charge sign is reverted.

We note, however, that the difference between complex conjugation and charge conjugation is relevant only in the realm of a theory that distinguishes mass signs. We can see this by covering the mass column in table I or II and noting that, in this case, those amplitudes are degenerate.

The probability density can be immediately obtained and is given by

$$\rho = \Psi^\dagger \sigma_3 \Psi = \Psi_{c1}\Psi, \tag{34}$$

where now, when permuting the amplitudes, we keep the sign. This should happen because each amplitude has a component related to a particle and another to an antiparticle (with the same sign of the charge).

The current or flux density can be easily obtained and is given by

$$\mathbf{j} = \frac{1}{2m}\left[\Psi_{c1}\Lambda\nabla\Psi - \left(\nabla\Psi_{c1}\right)\Lambda\Psi\right] - \frac{e\hbar}{mc}\mathbf{A}\Psi_{c1}\Lambda\Psi, \tag{35}$$

where

$$\Lambda = \left(\sigma_3 + i\sigma_2\right). \tag{36}$$

Before we go on with the study of particles with spin, it is interesting to consider particles with null charge. The usual interpretation denies these particles of being described by the KG formalism (at least if there is no interaction capable of distinguishing them). This is the case, for example, of the pion zero. Being a null charge particle, the associated charge density must be identically zero. These particles are then said to be their own antiparticles. We cannot say this in the present theory. Here, the pion zero might manifest itself with two masses of different signs that can be distinguished by a gravitational field. We are then faced with a pion zero and an antipion zero.

In the next section, we continue developing an analogous theory for half-spin particles.

4 Dirac's Theory with Negative Mass

We wish to develop a similar formalism for Dirac's equation as was done for Klein-Gordon's. As was already mentioned in the first papers of this series[1, 2, 3], we shall consider the second order Dirac's equation as the fundamental one rather than the first order equation.

We then depart from Dirac's second order equation

$$\frac{1}{c^2}\left(i\hbar\frac{\partial}{\partial t}-e\Phi\right)^2\left(\begin{array}{c}\varphi\\ \chi\end{array}\right)=\left[\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}\right)^2\mathbf{1}+m^2c^2\mathbf{1}+\right.$$

$$\left.+\frac{e\hbar}{c}\left(\begin{array}{cc}\sigma\cdot\mathbf{H} & \mathbf{0}\\ \mathbf{0} & \sigma\cdot\mathbf{H}\end{array}\right)-i\frac{e\hbar}{c}\left(\begin{array}{cc}\mathbf{0} & \sigma\cdot\mathbf{E}\\ \sigma\cdot\mathbf{E} & \mathbf{0}\end{array}\right)\right]\left(\begin{array}{c}\varphi\\ \chi\end{array}\right) \tag{37}$$

where φ, χ are two-component spinors, while $\mathbf{H}$ and $\mathbf{E}$ are the magnetic and electric field, respectively.

The expression for the probability density can be easily obtained and is given by

$$\rho_\lambda=\frac{1}{2\lambda mc^2}\left\{\left[\psi^\dagger\beta ih\frac{\partial\psi}{\partial t}-\left(i\hbar\frac{\partial\psi^\dagger}{\partial t}\right)\beta\psi\right]_+-2\lambda e\Phi\psi^\dagger\beta\psi\right\}, \tag{38}$$

where ψ is the four-component spinor

$$\psi=\left(\begin{array}{c}\varphi\\ \chi\end{array}\right) \tag{39}$$

and β is the usual spin parity operator in Dirac's representation. We are now in position to rewrite Dirac's formalism in the format given in the previous section. We will thus use a simile of the Feshbach-Villars decomposition applied to Dirac's second order equation.

Such a decomposition is attained if we define

$$\varphi_0=\left[\frac{\partial}{\partial t}+\frac{ie}{\hbar}\Phi\right]\varphi\ ;\ \chi_0=\left[\frac{\partial}{\partial t}+\frac{ie}{\hbar}\Phi\right]\chi \tag{40}$$

and

$$\left\{\begin{array}{l}\varphi_1=\frac{1}{2}\left(\varphi_0+\frac{i\hbar}{mc}\varphi\right)\\ \varphi_2=\frac{1}{2}\left(\varphi_0-\frac{i\hbar}{mc}\varphi\right)\end{array}\right.\ ;\ \left\{\begin{array}{l}\chi_1=\frac{1}{2}\left(\chi_0+\frac{i\hbar}{mc}\chi\right)\\ \chi_2=\frac{1}{2}\left(\chi_0-\frac{i\hbar}{mc}\chi\right)\end{array}\right., \tag{41}$$

where φ_1, φ_2, χ_1, χ_2 are two-component spinors. We are then led to the following equations:

$$\left(i\hbar\frac{\partial}{\partial t}-e\Phi\right)\varphi_1=\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}\right)^2\mathbf{1}-\frac{e\hbar}{2mc}\sigma\cdot\mathbf{H}\right](\varphi_1+\varphi_2)+$$

$$+mc^2\varphi_1+\frac{ie\hbar}{2mc}\sigma\cdot\mathbf{E}\,(\chi_1+\chi_2)\,; \tag{42}$$

$$\left(i\hbar\frac{\partial}{\partial t}-e\Phi\right)\varphi_2=\left[\frac{-1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}\right)^2\mathbf{1}+\frac{e\hbar}{2mc}\sigma\cdot\mathbf{H}\right](\varphi_1+\varphi_2)-$$

$$-\,mc^2\varphi_1-\frac{ie\hbar}{2mc}\sigma\cdot\mathbf{E}\,(\chi_1+\chi_2)\,; \tag{43}$$

$$\left(i\hbar\frac{\partial}{\partial t}-e\Phi\right)\chi_1=\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}\right)^2\mathbf{1}-\frac{e\hbar}{2mc}\sigma\cdot\mathbf{H}\right](\chi_1+\chi_2)+$$

$$+\,mc^2\chi_1+\frac{ie\hbar}{2mc}\sigma\cdot\mathbf{E}\,(\varphi_1+\varphi_2)\,; \tag{44}$$

$$\left(i\hbar\frac{\partial}{\partial t}-e\Phi\right)\chi_2=\left[\frac{-1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}\right)^2\mathbf{1}+\frac{e\hbar}{2mc}\sigma\cdot\mathbf{H}\right](\chi_1+\chi_2)-$$

$$-\,mc^2\chi_1-\frac{ie\hbar}{2mc}\sigma\cdot\mathbf{E}\,(\varphi_1+\varphi_2)\,. \tag{45}$$

These equations, togheter with their complex conjugate, cover all the possibilities we expect from Nature when leting for the existence of negative masses (Table 2).

Defining the eight-component spinor

$$\Psi=\begin{bmatrix}\varphi_1\\ \varphi_2\\ \chi_1\\ \chi_2\end{bmatrix}\ ;\ \varphi_i=\begin{bmatrix}\varphi_{i1}\\ \varphi_{i2}\end{bmatrix}\ ;\ \chi_i=\begin{bmatrix}\chi_{i1}\\ \chi_{i2}\end{bmatrix}\ , \tag{46}$$

the matrices

$$\Sigma_1=\begin{bmatrix}\mathbf{0}&\mathbf{+1}&\mathbf{0}&\mathbf{0}\\ \mathbf{+1}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{+1}\\ \mathbf{0}&\mathbf{0}&\mathbf{+1}&\mathbf{0}\end{bmatrix}\ ;\ \Sigma_2=\begin{bmatrix}\mathbf{0}&\mathbf{-i}&\mathbf{0}&\mathbf{0}\\ \mathbf{i}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{-i}\\ \mathbf{0}&\mathbf{0}&\mathbf{i}&\mathbf{0}\end{bmatrix} \tag{47}$$

$$\Sigma_3=\begin{bmatrix}\mathbf{+1}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{-1}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{+1}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{-1}\end{bmatrix}\ ;\ \alpha_1=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{1}\\ \mathbf{0}&\mathbf{0}&\mathbf{1}&\mathbf{0}\\ \mathbf{0}&\mathbf{1}&\mathbf{0}&\mathbf{0}\\ \mathbf{1}&\mathbf{0}&\mathbf{0}&\mathbf{0}\end{bmatrix}; \tag{48}$$

$$\alpha_2=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{-i}\\ \mathbf{0}&\mathbf{0}&\mathbf{+i}&\mathbf{0}\\ \mathbf{0}&\mathbf{-i}&\mathbf{0}&\mathbf{0}\\ \mathbf{+i}&\mathbf{0}&\mathbf{0}&\mathbf{0}\end{bmatrix}\ ;\ \alpha_3=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{+1}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{-1}\\ \mathbf{+1}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{-1}&\mathbf{0}&\mathbf{0}\end{bmatrix} \tag{49}$$

and

$$\beta=\begin{bmatrix}\mathbf{+1}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{+1}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{-1}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{-1}\end{bmatrix}, \tag{50}$$

where each element is a 2×2 matrix, we can write the above system of equations as:

$$\left(i\hbar\frac{\partial}{\partial t}-e\Phi\right)\Psi=\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}\right)^2\mathbf{1}-\frac{e\hbar}{2mc}\sigma\cdot\mathbf{H}\right]\left(\Sigma_3+i\Sigma_2\right)\Psi+$$

$$+\,mc^2\Sigma_3\Psi+\frac{ie\hbar}{2mc}\sigma\cdot\mathbf{E}\left(\alpha_3+i\alpha_2\right)\Psi. \tag{51}$$

It is then easy to show that

$$\Psi_{c1}=i\beta\sigma_2\Psi^* \tag{52}$$

is a solution of

$$\left(i\hbar\frac{\partial}{\partial t}+e\Phi\right)\Psi_{c1}=\left[\frac{-1}{2m}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}\mathbf{A}\right)^2\mathbf{1}-\frac{e\hbar}{2mc}\sigma\cdot\mathbf{H}\right]\left(\Sigma_3+i\Sigma_2\right)\Psi_{c1}-$$

$$-\,mc^2\Sigma_3\Psi_{c1}+\frac{ie\hbar}{2mc}\sigma\cdot\mathbf{E}\left(\alpha_3+i\alpha_2\right)\Psi_{c1} \tag{53}$$

which is the same equation solved by Ψ with the signs of the mass and the charge inverted, but with the same parity.

We can also show that

$$\Psi_{c2}=\Psi^\dagger\Sigma_3 i\alpha_3\beta \tag{54}$$

is a solution of

$$\Psi_{c2}\left(i\hbar\frac{\partial}{\partial t}+e\Phi\right)=\Psi_{c2}\left(\Sigma_3+i\Sigma_2\right)\left[\frac{-1}{2m}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}\mathbf{A}\right)^2\mathbf{1}-\frac{e\hbar}{2mc}\sigma\cdot\mathbf{H}\right]-$$

$$-\,mc^2\Psi_{c2}\Sigma_3-\frac{ie\hbar}{2mc}\sigma\cdot\mathbf{E}\Psi_{c2}\left(\alpha_3+i\alpha_2\right) \tag{55}$$

which is similar to the one solved by Ψ with the signs of the mass, the charge and the parity inverted, while keeping the signs of the spins.

Both the above amplitudes are candidates to represent antiparticles of Ψ since we have used, until now, only the criterion of the mass and charge signs. We might write them explicitly as

$$\Psi=\begin{bmatrix}\varphi_{11}(+,+)\\ \varphi_{12}(+,+)\\ \varphi_{21}(-,+)\\ \varphi_{22}(-,+)\\ \chi_{11}(+,+)\\ \chi_{12}(+,+)\\ \chi_{21}(-,+)\\ \chi_{22}(-,+)\end{bmatrix}\;\Rightarrow\;\Psi_{c1}=\begin{bmatrix}+\varphi_{12}^*(-,-)\\ -\varphi_{11}^*(-,-)\\ +\varphi_{22}^*(+,-)\\ -\varphi_{21}^*(+,-)\\ -\chi_{12}^*(-,-)\\ +\chi_{11}^*(-,-)\\ -\chi_{22}^*(+,-)\\ +\chi_{21}^*(+,-)\end{bmatrix}\;;\;\Psi_{c2}=\begin{bmatrix}+\chi_{11}^*(-,-)\\ +\chi_{12}^*(-,-)\\ +\chi_{21}^*(+,-)\\ +\chi_{22}^*(+,-)\\ -\varphi_{11}^*(-,-)\\ -\varphi_{12}^*(-,-)\\ -\varphi_{21}^*(+,-)\\ -\varphi_{22}^*(+,-)\end{bmatrix}, \tag{56}$$

where we also show, inside parenthesis, the signs of the mass and the charge related to each component of the spinors (we took the transpose of the line-spinor). This arrangement shows more clearly what relation particles exhibit with antiparticles by means of the above mentioned functions.

We now define the element

$$u^{(P,+)}_{0\uparrow(+)}, \tag{57}$$

as the eight-component spinor where: the index zero denotes that we are in the rest frame, the up arrow indicates the spin up (upon action of operator Σ_3), the pair $(P,+)$ implies that we have a particle with positive charge and the lower index $(+)$ denotes that the spin parity is positive (upon action of operator β). With the usual eight canonical basis vectors, $\mathbf{e}_i, i = 1..8$ that are extensions of the two-dimensional KG case, we can write the eight distinct possibilities for Ψ as

$$u^{(P,+)}_{0\uparrow(+)} = \mathbf{e}_1 e^{-iE_p\tau/\hbar} \quad ; \quad u^{(P,+)}_{0\downarrow(+)} = \mathbf{e}_2 e^{-iE_p\tau/\hbar};$$

$$u^{(A,+)}_{0\downarrow(+)} = \mathbf{e}_3 e^{+iE_p\tau/\hbar} \quad ; \quad u^{(A,+)}_{0\uparrow(+)} = \mathbf{e}_4 e^{+iE_p\tau/\hbar}; \tag{58}$$

$$v^{(P,+)}_{0\uparrow(-)} = \mathbf{e}_5 e^{-iE_p\tau/\hbar} \quad ; \quad v^{(P,+)}_{0\downarrow(-)} = \mathbf{e}_6 e^{-iE_p\tau/\hbar};$$

$$v^{(A,+)}_{0\downarrow(-)} = \mathbf{e}_7 e^{+iE_p\tau/\hbar} \quad ; \quad v^{(A,+)}_{0\uparrow(-)} = \mathbf{e}_8 e^{+iE_p\tau/\hbar}, \tag{59}$$

where

$$E_p = m_0 c^2. \tag{60}$$

With the correspondence (55) between particle and antiparticle spinors, we can write the spinors for Ψ_{c1}

$$\mu^{(A,-)}_{0\uparrow(+)} = \mathbf{e}_1 e^{+iE_p\tau/\hbar} \quad ; \quad \mu^{(A,-)}_{0\downarrow(+)} = \mathbf{e}_2 e^{+iE_p\tau/\hbar};$$

$$\mu^{(P,-)}_{0\downarrow(+)} = \mathbf{e}_3 e^{-iE_p\tau/\hbar} \quad ; \quad \mu^{(P,-)}_{0\uparrow(+)} = \mathbf{e}_4 e^{-iE_p\tau/\hbar}, \tag{61}$$

$$\nu^{(A,-)}_{0\uparrow(-)} = \mathbf{e}_5 e^{+iE_p\tau/\hbar} \quad ; \quad \nu^{(A,-)}_{0\downarrow(-)} = \mathbf{e}_6 e^{+iE_p\tau/\hbar};$$

$$\nu^{(P,-)}_{0\downarrow(-)} = \mathbf{e}_7 e^{-iE_p\tau/\hbar} \quad ; \quad \nu^{(P,-)}_{0\uparrow(-)} = \mathbf{e}_8 e^{-iE_p\tau/\hbar}, \tag{62}$$

while for Ψ_{c2}

$$\omega^{(A,-)}_{0\uparrow(-)} = \mathbf{e}_5 e^{+iE_p\tau/\hbar} \quad ; \quad \omega^{(A,-)}_{0\downarrow(-)} = \mathbf{e}_6 e^{+iE_p\tau/\hbar};$$

$$\omega^{(P,-)}_{0\downarrow(-)} = \mathbf{e}_7 e^{-iE_p\tau/\hbar} \quad ; \quad \omega^{(P,-)}_{0\uparrow(-)} = \mathbf{e}_8 e^{-iE_p\tau/\hbar}, \tag{63}$$

$$\eta^{(A,-)}_{0\uparrow(-)} = \mathbf{e}_1 e^{+iE_p\tau/\hbar} \quad ; \quad \eta^{(A,-)}_{0\downarrow(-)} = \mathbf{e}_2 e^{+iE_p\tau/\hbar};$$

$$\eta^{(P,-)}_{0\downarrow(-)} = \mathbf{e}_3 e^{-iE_p\tau/\hbar} \quad ; \quad \eta^{(P,-)}_{0\uparrow(-)} = \mathbf{e}_4 e^{-iE_p\tau/\hbar}. \tag{64}$$

We also get the following relations between the antiparticle spinors

$$\omega_{0\uparrow} = \nu_{0\downarrow} \ ; \omega_{0\downarrow} = \nu_{0\uparrow} \text{ and } \eta_{0\uparrow} = \mu_{0\downarrow} \ ; \ \eta_{0\downarrow} = \mu_{0\uparrow}; \tag{65}$$

together with the spin parity relations

$$\begin{cases} \beta u_0 = +u_0 \\ \beta v_0 = -v_0 \end{cases} ; \begin{cases} \beta \mu_0 = +\mu_0 \\ \beta \nu_0 = -\nu_0 \end{cases} ; \begin{cases} \beta \omega_0 = -\omega_0 \\ \beta \eta_0 = +\eta_0 \end{cases} . \tag{66}$$

These results can also be compared with those obtained using the linear Dirac's equation[7].

We give the annihilation relations in Table 3.

We can now obtain the expression for the densities of probability and current in the present formalism. This is a straightforward extension of what was done in the KG formalism. We get equation

$$\frac{\partial}{\partial t}\left(\Psi_{c2}\Psi\right) + \nabla \cdot \left\{ \frac{1}{2m}\left[\Psi_{c2}\Lambda\nabla\Psi - \left(\nabla\Psi_{c2}\right)\Lambda\Psi\right] - \frac{e\mathbf{A}}{mc}\Psi_{c2}\Lambda\Psi \right\} = 0, \tag{67}$$

where

$$\Lambda = \left(\Sigma_3 + i\Sigma_2\right). \tag{68}$$

Equation (67) then implies that

$$\rho = \Psi_{c2}\Psi \tag{69}$$

and

$$\mathbf{j} = \frac{1}{2m}\left[\Psi_{c2}\Lambda\nabla\Psi - \left(\nabla\Psi_{c2}\right)\Lambda\Psi\right] - \frac{e\mathbf{A}}{mc}\Psi_{c2}\Lambda\Psi. \tag{70}$$

We can write the expression for the density only in terms of Ψ

$$\rho = \Psi^{\dagger}\Sigma_3 i\alpha_3\beta\Psi, \tag{71}$$

which gives, considering (67), the conservation equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0. \tag{72}$$

With straightforward calculations we can show that one might write the probability density, using the Ψ components, as

$$\rho = Im\left[\sum_{ij}^{2}\varphi_i^{\dagger}\chi_j\right]. \tag{73}$$

Finally, we shall comment the results of Table 3. Until now, we do not impose any constraint on the parity of annihilating particles. This degree of freedom leads to the possibility represented by the first column of Table 3, where particles

and antiparticles with the same parity annihilate each other. The problem with this annihilation process is that physicists have not found, until now, any spin zero (longitudinal) photon. There are indeed strong arguments against their existence. We could then postulate that particles and antiparticles should have the spin parity (if any) also reverted. However, we avoid to assert this for the moment, since we are here interested in uncovering worlds, not in hiding them.

5 Conclusions

We have then succeeded in showing that our conjecture can be accommodated into the formal apparatus of the special relativistic quantum mechanics.

When Dirac's theory, based on its first order equation, revealed the antiparticle (as were defined), many physicists were delighted with the symmetries it has brought[4, 8]. For each element with mass of a given value, Nature does not distinguish them by giving different charges. So, the massive proton with positive charge shall have its negative charge counterpart. Each particle has its antiparticle defined by its charge mirror.

This work takes this approach to its utmost limits. Each particle has its antiparticle defined by its mirror world, where both charge and mass signs are reverted. Also, there is no particle being its own antiparticle in the sense that only entities with opposite mass sign might annihilate each other. In this case, pions zero are annihilated by antipions zero (both, of course, might decay spontaneously).

The vacuum that emerges is not a filled structure in which every point of the real space is occupied with an infinitude of antiparticles. This picture can be avoided while keeping the important property of vacuum polarization. Moreover, contrarily to the usual interpretation, the present theory treats particles and antiparticles in a totally symmetrical way [(5)]. We shall also stress, considering the present conjecture, that the gravitational field is highly capable of polarizing the vacuum. This property will become relevant in the second paper of this series.

This theory does not claim for a strict inertial mass conservation law. This is because for mass we have Einstein's equation, $E = mc^2$, which distinguishes mass from charge with respect to conservative behavior. If we also admit, following the discussion at the end of the last section, that creation and annihilation processes shall conserve parity, then we place parity, aside from the charge, as a fundamental property of Nature.

The possible existence of negative masses have far reaching cosmological consequences that will be addressed in a future paper.

The arguments above, about the higher symmetry of Nature introduced by the concept of negative masses, cannot, of course, prove the conjecture. They fail to have any necessity character. They are just a metaphysical constraint we wish to impose upon Nature. The final word will be with the experimental physicists.

This formidable task is being presently carried on by several experiments[9].

Clearly, in the realm of special relativistic quantum mechanics, fixing mass signs is an ad hoc postulate, as we stated in the last paragraph. The next paper of this series will show, however, that the general relativistic quantum mechanical theory derived in paper II of this series supports this conjecture.

A Negative Densities

When studying the KG formalism, we are faced with a striking fact. While the amplitudes in (7) indicate that we should expect both positive and negative energy densities, the energy density obtained from the energy-momentum tensor is always positive. Moreover, since we sustain[1, 2, 3] that relativistic quantum mechanics can be derived from classical relativity and statistics, where we impose the positive character of the energy, it is higly desirable to clarify this apparent paradox.

We will show in this appendix that this paradoxical situation can be easily clarified. We will use the formalism already developed[1, 2, 3] that enables us to go from Liouville's equation to the equation for the density function. From the analysis of what is happening in phase space, it will be easier to understand this property of the KG equation. In fact, it will be shown that this "pathology" is also present in the non-relativistic Schrödinger equation. We will thus present both the non-relativistic and relativistic calculations to make our discussion clearer.

We have shown[1, 2, 3] that all non-relativistic and relativistic quantum mechanics could be obtained from the classical Liouville's equation

$$\frac{dF_n\left(\mathbf{x},\mathbf{p};t\right)}{dt}=0\ ;\ \frac{dF_r\left(x,p\right)}{d\tau}=0, \tag{74}$$

where $\mathbf{x}$ and $\mathbf{p}$ are the position and momentum vectors, x and p are the related four-vectors, τ is the proper time and F_n and F_r are the non-relativistic and relativistic joint probability densities, respectively. This was accomplished using the Infinitesimal Wigner-Moyal Transformations

$$\rho_n^{(d)}\left(\mathbf{x}-\frac{\delta\mathbf{x}}{2},\mathbf{x}+\frac{\delta\mathbf{x}}{2};t\right)=\int F_n\left(\mathbf{x},\mathbf{p};t\right)\exp\left(\frac{i}{\hbar}\mathbf{p}\cdot\delta\mathbf{x}\right)d^3p \tag{75}$$

and

$$\rho_r^{(d)}\left(x-\frac{\delta x}{2},x+\frac{\delta x}{2}\right)=\int F_r\left(x,p;t\right)\exp\left(\frac{i}{\hbar}p^{\alpha}\delta x_{\alpha}\right)d^4p, \tag{76}$$

where ρ_n and ρ_r are the non-relativistic and relativistic density functions, respectively. We also assumed as an axiom that Newton's equation, and its special relativistic counterpart, are valid

$$\frac{d\mathbf{x}}{dt}=\frac{\mathbf{p}}{m}\ ;\ \frac{dx^{\alpha}}{d\tau}=\frac{p^{\alpha}}{m}; \tag{77}$$

to obtain the equations

$$\frac{-\hbar^2}{2m}\frac{\partial^2\rho_n^{(d)}}{\partial\mathbf{x}\partial\left(\delta\mathbf{x}\right)}=i\hbar\frac{\partial\rho_n^{(d)}}{\partial t}\ ;\ \hbar^2\frac{\partial^2\rho_r^{(d)}}{\partial x^\alpha\partial\left(\delta x_\alpha\right)}=0. \tag{78}$$

These equations, we showed, can be taken into the Schrödinger's and Klein-Gordon's equations (in the absence of external forces and spin, for simplicity)

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi_n}{\partial\mathbf{x}^2}=i\hbar\frac{\partial\psi_n}{\partial t}\ ;\ \left(\hbar^2\Box-m^2\right)\psi_r=0, \tag{79}$$

where ψ_n and ψ_r are the non-relativistic and relativistic probability amplitudes, when we use the property that δx represents an infinitesimal variation and that the expansions

$$\rho_{n(+)}^{(d)}\left(\mathbf{x}-\frac{\delta\mathbf{x}}{2},\mathbf{x}+\frac{\delta\mathbf{x}}{2};t\right)=\psi_n^*\left(\mathbf{x}-\frac{\delta\mathbf{x}}{2};t\right)\psi_n\left(\mathbf{x}+\frac{\delta\mathbf{x}}{2};t\right) \tag{80}$$

and

$$\rho_{r(+)}^{(d)}\left(x-\frac{\delta x}{2},x+\frac{\delta x}{2}\right)=\psi_r^*\left(x-\frac{\delta x}{2}\right)\psi_r\left(x+\frac{\delta x}{2}\right) \tag{81}$$

might be performed.

From these expressions, we can define the 3 and 4-momentum operators by means of the expressions for their expectation values

$$\langle\mathbf{p}\rangle=\lim_{\delta\mathbf{x}\to0}\frac{\hbar}{i}\frac{\partial}{\partial\left(\delta\mathbf{x}\right)}\int\rho_{n(+)}^{(d)}\left(\mathbf{x}-\frac{\delta\mathbf{x}}{2},\mathbf{x}+\frac{\delta\mathbf{x}}{2};t\right)d^3x; \tag{82}$$

$$\langle p\rangle=\lim_{\delta x\to0}\frac{\hbar}{i}\frac{\partial}{\partial\left(\delta x\right)}\int\rho_{r(+)}^{(d)}\left(x-\frac{\delta x}{2},x+\frac{\delta x}{2};t\right)d^4x. \tag{83}$$

It is noteworthy that we have, however, a freedom of choice in expressions (80) and (81). We could equally well have chosen

$$\rho_{n(-)}^{(d)}\left(\mathbf{x}-\frac{\delta\mathbf{x}}{2},\mathbf{x}+\frac{\delta\mathbf{x}}{2};t\right)=\psi_n^*\left(\mathbf{x}+\frac{\delta\mathbf{x}}{2};t\right)\psi_n\left(\mathbf{x}-\frac{\delta\mathbf{x}}{2};t\right)=\rho_{n(+)}^{(d)\dagger} \tag{84}$$

and

$$\rho_{r(-)}^{(d)}\left(x-\frac{\delta x}{2},x+\frac{\delta x}{2}\right)=\psi_r^*\left(x+\frac{\delta x}{2}\right)\psi_r\left(x-\frac{\delta x}{2}\right)=\rho_{r(+)}^{(d)\dagger} \tag{85}$$

that is equivalent to the change $\psi\leftrightarrow\psi^*, i=n,r$. It is easy to see that, with this new definition, we get

$$\langle\mathbf{p}\rangle\to-\langle\mathbf{p}\rangle\ ;\ \langle p\rangle\to-\langle p\rangle\,. \tag{86}$$

We can interpret these results as representing, in the non-relativistic case, a problem where the particle travels back in space. In the relativistic case it

can be understood as if the particle travels back in space-time (with negative momentum and energy).

However, if we still want to have an adequate definition of three and four-momentum, as given by (82) and (83), we shall redefine the 3- and 4-momentum mean values as

$$\langle \mathbf{p} \rangle = \lim_{\delta \mathbf{x} \to 0} -\frac{\hbar}{i} \frac{\partial}{\partial (\delta \mathbf{x})} \int \rho_{n(-)}^{(d)} \left(\mathbf{x} - \frac{\delta \mathbf{x}}{2}, \mathbf{x} + \frac{\delta \mathbf{x}}{2}; t \right) d^3 x; \tag{87}$$

$$\langle p \rangle = \lim_{\delta x \to 0} -\frac{\hbar}{i} \frac{\partial}{\partial (\delta x)} \int \rho_{r(-)}^{(d)} \left(x - \frac{\delta x}{2}, x + \frac{\delta x}{2}; t \right) d^4 x. \tag{88}$$

which give the operators

$$\mathbf{p}_{op} = \lim_{\delta \mathbf{x} \to 0} -\frac{\hbar}{i} \frac{\partial}{\partial (\delta \mathbf{x})} \; ; \; p_{op} = \lim_{\delta x \to 0} -\frac{\hbar}{i} \frac{\partial}{\partial (\delta x)}. \tag{89}$$

In general, we have

$$\mathbf{p}_{op} = \lim_{\delta \mathbf{x} \to 0} \lambda \frac{\hbar}{i} \frac{\partial}{\partial (\delta \mathbf{x})} \; ; \; p_{op} = \lim_{\delta x \to 0} \lambda \frac{\hbar}{i} \frac{\partial}{\partial (\delta x)}, \tag{90}$$

where λ has the same definition given in the main text, when acting upon $\rho_{i,\lambda}, i = n, r$. This assures that the energy and momentum have the correct sign when calculated by expressions (82-83) or (87-88).

It is important to stress that equations (78) do not depend upon λ, since they are quadratic in the considered quantities (energy and momentum for KG's and momentum for Schrödinger's). This justifies that the energy density obtained from the energy-momentum tensor is always positive. In the same way, the energy in Schrödinger's equation is not affected, since the time does not enter into the non-relativistic transformation (75).

With the conventions (90), the relativistic energy density[1, 2, 3] can be written, in the absence of electromagnetic fields, as

$$p_\lambda^0 (x) = \frac{i\hbar}{2} \lambda \left[\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right] = \frac{i\hbar}{2} \left[\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right]_+ , \tag{91}$$

which is always positive. If we impose the possibility of negative masses for the complex conjugate amplitudes, the probability density can be written as

$$\rho_\lambda (x) = \frac{i\hbar}{2\lambda m c^2} \left[\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right]_+ , \tag{92}$$

where λ now comes from the mass sign.

Let us consider now the relativistic situation when electromagnetic fields are present. The energy density is now given by

$$p_\lambda^0 (x) = \frac{i\hbar}{2} \left[\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right]_+ - \lambda e \Phi \psi^* \psi, \tag{93}$$

where e is the particle charge and Φ is the scalar electromagnetic potential. The parameter λ appears multiplying the charge, since in the equation that φ^* solves, the charge changes sign. The probability density can be written as

$$\rho_\lambda(x) = \frac{i\hbar}{2\lambda mc^2}\left[\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right]_+ - \frac{e}{mc^2}\Phi\psi^*\psi, \tag{94}$$

as was presented in the main text.

It is important to note that (94) is a density that represents particles (positive mass, given by ψ) and antiparticles (negative mass, given by ψ^*) with energy and momentum positive, as stressed in the main text (9). We thus obtain results in agreement with those obtained with the energy-momentum tensor.

The final expression for the density with the normalization

$$\int \rho_\lambda(x)\, d^3x = \lambda \tag{95}$$

implies that, without fields, the worlds of particles and antiparticles fall apart. This is a very important property; it allows us to avoid the vacuum picture that emerges from Dirac's theory based in his first order equation. This theory is automatically prevented from a radiative catastrophe.

The above calculations also allow us to understand the picture of particle flow. A positive mass particle, with negative energy and momentum traveling backward on time, is equivalent to a negative mass antiparticle, with positive energy and momentum, traveling in the usual time direction. These conventions just reflect equations (10-12) and agree with the signs of the velocity as being opposite for particles and antiparticles. These considerations will play an important role in the next paper of this series.

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mass	**charge**	**amplitude**
+	+	χ_1
+	-	$\chi_2^\dagger$
-	-	$\chi_1^\dagger$
-	+	χ_2

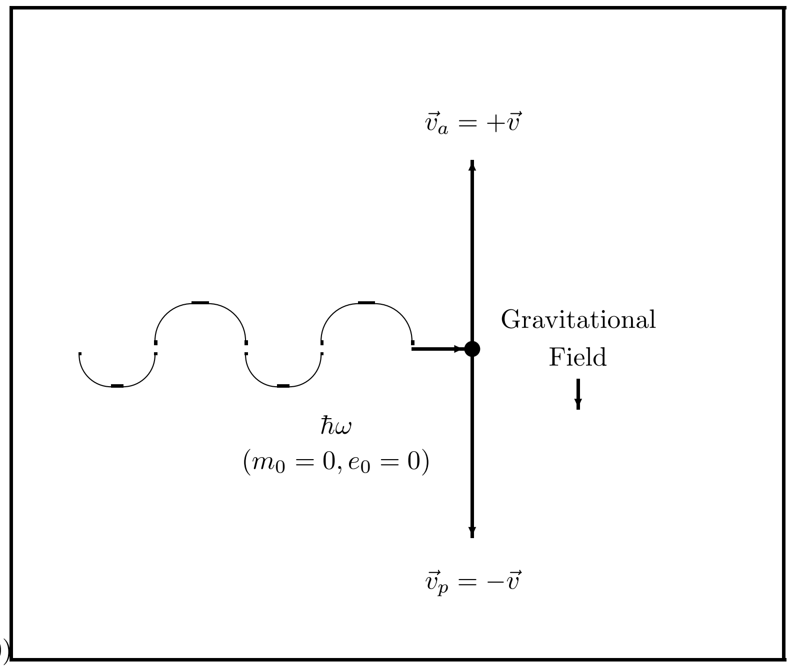
Table 1: Possible combinations of mass and charge signals allowed by Nature for spinless particles.

mass	**charge**	**spin**	**amplitude**
+	+	$\uparrow$	ϕ_1
+	+	$\downarrow$	χ_1
+	-	$\uparrow$	$\chi_2^\dagger$
+	-	$\downarrow$	$\phi_2^\dagger$
-	+	$\uparrow$	ϕ_2
-	+	$\downarrow$	χ_2
-	-	$\uparrow$	$\chi_1^\dagger$
-	-	$\downarrow$	$\phi_1^\dagger$

Table 2: Possible combinations allowed by Nature for particles with spin.

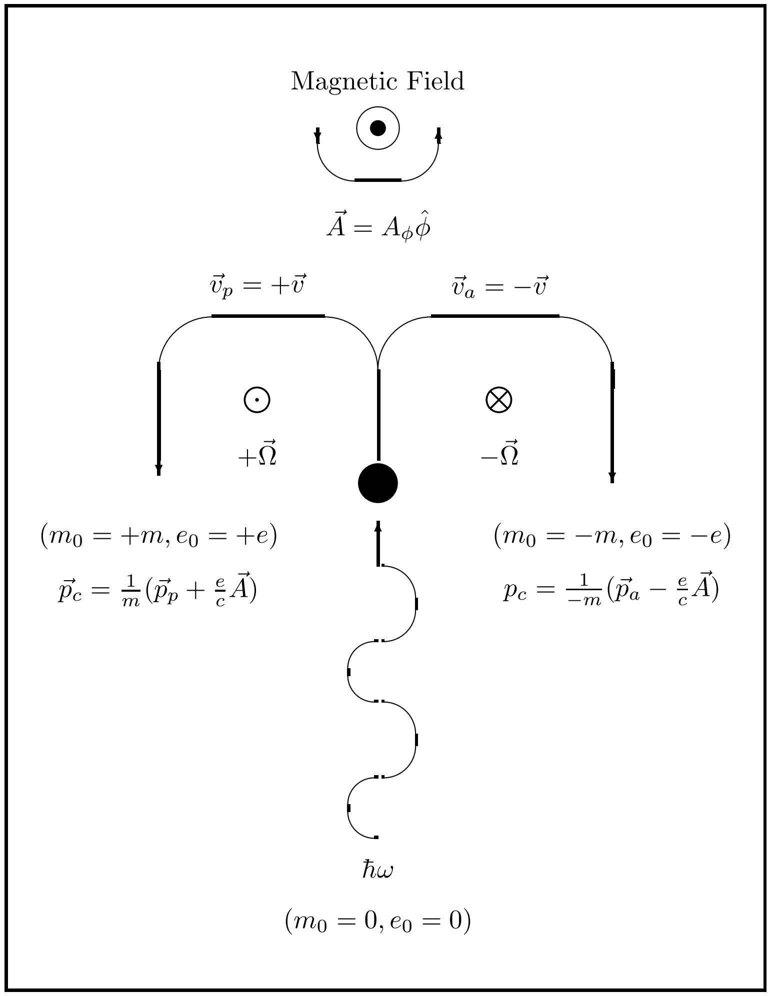
$\mathbf{\Psi}$	$\mathbf{\Psi}_{c1}(\hbar\omega)$	$\mathbf{\Psi}_{c2}(\hbar\omega)$
$u^{(}_{0\uparrow(+)}P,+)$	$\mu^{(A,-)}_{0\downarrow(+)}(0)$	$\omega^{(A,-)}_{0\uparrow(-)}(+1)$
$u^{(}_{0\downarrow(+)}P,+)$	$\mu^{(A,-)}_{0\uparrow(+)}(0)$	$\omega^{(A,-)}_{0\downarrow(-)}(-1)$
$u^{(}_{0\downarrow(+)}A,+)$	$\mu^{(P,-)}_{0\uparrow(+)}(0)$	$\omega^{(P,-)}_{0\downarrow(-)}(-1)$
$u^{(}_{0\uparrow(+)}A,+)$	$\mu^{(P,-)}_{0\downarrow(+)}(0)$	$\omega^{(P,-)}_{0\uparrow(-)}(+1)$
$v^{(}_{0\uparrow(-)}P,+)$	$\nu^{(A,-)}_{0\downarrow(-)}(0)$	$\eta^{(A,-)}_{0\uparrow(+)}(+1)$
$v^{(}_{0\downarrow(-)}P,+)$	$\nu^{(A,-)}_{0\uparrow(-)}(0)$	$\eta^{(A,-)}_{0\downarrow(+)}(-1)$
$v^{(}_{0\downarrow(-)}A,+)$	$\nu^{(P,-)}_{0\uparrow(-)}(0)$	$\eta^{(P,-)}_{0\downarrow(+)}(-1)$
$v^{(}_{0\uparrow(-)}A,+)$	$\nu^{(P,-)}_{0\downarrow(-)}(0)$	$\eta^{(P,-)}_{0\uparrow(+)}(+1)$

Table 3: Possible combinations allowed by Nature for particles with spin.



(102.00,86.00)

Figure 1: Particle-antiparticle trajectories in the presence of a gravitational field.



(101.00,132.00)

Figure 2: Particle-Antiparticle trajectories in the presence of a homogeneous magnetic field.